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Random PERT: application to physical activity/sports programs

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Abstract This paper illustrates the variety of PERT technique known as random PERT. The aim of this technique is to help plan the duration of activities, something which can be particularly difficult in psychosocial programs. Thus, this task is often carried out by experts, who know that there are many events which may modify the proposed calendar. The paper includes an empirical illustration of random PERT applied to a physical activity/sports program for elderly people.

Keywords Random PERT · Physical Activity · Sport Programs

1 Introduction

Since the 1950s the strategy known as PERT has, along with other critical path techniques, been regarded as useful for planning and scheduling various kinds of program. An essential feature of the technique is uncertainty, and therefore it has adopted a probabilistic approach, in contrast to the deterministic approach taken by the closely related technique of CPM (Pillai and Tiwari 1995). In a traditional PERT analysis, the main objective was to schedule a project assuming deterministic durations. However, each one of the various activities requires the use of available resources (equipment, materials, etc.) and, as the resource combinations themselves also imply a cost, this may create logistical problems (Sánchez-Algarra and Anguera, in press).

Indeed, one of the most controversial issues in the history of PERT has been the approach to activity times (Sasieni 1986; Littlefield and Randolph 1987, 1991; Gallagher 1987; Farnum and Stanton 1987; Golenko-Ginzburg 1988, 1989; Sculli 1989; Chae 1990; Chae and Kim 1990; Rhiel 1990; Keefer and Verdini 1993; Somarajan et al. 1992; Williams 1995;

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Kamburowski 1997). In the specific case of social intervention programs, especially those concerning physical activity, numerous difficulties must often be overcome in order to decide realistically on the times for different activities, whether these be concurrent or diachronic; furthermore, a schedule must be explicitly set out in such a way that it enables the proposed intervention plan to be organized on the basis of the interrelationship between points in time and activities (Collantes 1982; Kerzner 1998; Prado 1988; Yu 1989; Alberich 1995; Muscatello 1988; Sánchez-Algarra and Anguera 1993; Hernández Mendo and Anguera 2001).

The recent use of computational optimization techniques has led to genetic algorithms being proposed (Feng et al. 1997; Chau et al. 1997), although during project implementation certain unknown variables may affect the planned durations and, therefore, the costs. This affects the temporal proposal of PERT, a problem which only recently has been resolved by Azaron et al. (in press); these authors, whose work is based on the earlier proposal by Kulkarni and Adlakha (1986), assume that the mean duration of each activity is a non-increasing function and that the direct cost of each activity is a non-decreasing function of the assigned resources.

2 Random PERT

When using the PERT technique it is necessary to calculate what are termed the earliest times (optimistic times) and latest times (pessimistic times). The use of random PERT in any social intervention program is especially recommended when the duration of activities is not known very precisely. Therefore, it is assumed that the durations are *random variables,* for which the probability distributions are known, and hence that the activity durations are random variables that follow beta probability laws. The density function of a random variable t with a beta probability distribution in an interval $[a, b]$ is as follows:

$$\begin{aligned} f(t) &= 0 \quad \text{for } t < a \\ f(t) &= 0 \quad \text{for } t > a \\ f(t) &= K(t-a)^{\alpha}(b-t)^{\beta} \quad \text{for } a < t < b \end{aligned}$$

where K is a constant which depends on the values of a and b, and on the parameters α and β.

The above expression represents a family of beta density curves which will be asymmetrical toward the right if

$$\frac{(a+b)}{2} > m$$

and toward the left if

$$\frac{(a+b)}{2} < m$$

Moreover, the curves are not asymptotic with respect to the abscissa axis, but rather cross it at the end points of the distribution, a and b. However, they resemble, to an extent, the curves of a normal distribution.

The mean and variance of these distributions will be:

$$D = \frac{a + (\alpha + \varphi)m + b}{\alpha + \varphi + 2} \tag{1}$$

$$v^2 = \frac{(b-a)^2(\alpha+1)(\varphi+1)}{(\alpha+\varphi+2)^2(\alpha+\varphi+3)} \tag{2}$$

In order to estimate which of the possible curves shows the best fit to the representation of the activity durations, it is assumed, for the PERT technique, that the standard deviation of the distribution is one sixth of the total, such that:

$$v = \frac{1}{6}(b-a) \tag{3}$$

which fits beta distributions adequately. If we introduce this condition we will establish a single beta distribution, in other words, we will determine single values for α and ϕ from the end values a and b and the mode value m of the distribution. The mode of the distribution can be calculated by deriving the density function with respect to t and making the result equal to zero:

$$\alpha(b-t) - \varphi(t-a) = 0 \tag{4}$$

The mode is obtained by finding the value of t from the previous equation and substituting t by m:

$$m = \frac{a\varphi - b\alpha}{\alpha+\varphi} \tag{5}$$

If we consider the standard deviation Eq. 3, the expression of variance can be taken as:

$$\frac{(\alpha+1)(\varphi+1)}{(\alpha+\varphi+2)^2(\alpha+\varphi+3)} = \frac{1}{36} \tag{6}$$

Solving the set of equations formed by (6) and (3) yields the values of α and φ which determine the curve that must be used in the PERT technique. The values of the mean and variance of t will also be estimated as the activity duration. The values of α and φ that are normally used are as follows:

$$\alpha = 2+\sqrt{2}, \quad \varphi = 2-\sqrt{2}, \quad \text{if } m \geq \frac{a+b}{2} \tag{7a}$$

Or alternatively:

$$\alpha = 2-\sqrt{2}, \quad \varphi = 2+\sqrt{2}, \quad \text{if } m \leq \frac{a+b}{2} \tag{7b}$$

Substituting these values in the expressions of the mean (1) and variance (6) of the beta distribution yields:

$$D = \frac{a+4m+b}{6} \tag{8}$$

and also,

$$v^2 = \frac{(b-a)^2}{36} \tag{9}$$

which leads us back to the previous formula of activity time and enables its justification. However, there are actually no strong grounds for assuming that the distribution of activity durations takes a beta form. The statistical assumptions used in the PERT technique may generate absolute errors of up to 33% of the total for the mean of the duration and 17% for its standard deviation.

The above equations can provide complementary information which is useful in scheduling and controlling programs and, in particular, it is possible to determine the 'probability' of adhering to the estimates proposed by PERT on the basis of the earliest times.

If we call the activities $i, j, \ldots,$ where a_i, m_i, and b_i, are the three estimates of activity duration i, and we take a random variable that measures the duration of an activity and which belongs to the project's critical path, the mean and variance of this variable ξ_i will be given by:

$$D_i = \frac{a_i + 4m_i + b_i}{6} \tag{10}$$

$$v_i^2 = \left(\frac{b_i - a_i}{6}\right)^2 \tag{11}$$

If we define η as a new random variable, such that:

$$\eta = \xi_1 + \xi_2 + \cdots + \xi_i + \cdots + \xi_n = \sum \xi_i \tag{12}$$

then, following the central limit theorem, the distribution of the sum of n random variables, which are distributed in the same way and independently, converges into a normal distribution whose mean and variance is the sum of the means and variances of the n variables, when n tends towards infinity.

$$\eta \; \alpha \; N\left[M = \sum D_i;\, V^2 = \sum V_i^2\right] \tag{13}$$

Thus, the duration of a program will be a normal variable whose parameters will be the sum of the different critical path activities. In order to determine the probability of completing a project in time T, it is necessary to calculate:

$$P\,(\eta \leq T) = F\,(T) \tag{14}$$

which corresponds to the function:

$$f\left(\underset{-\infty \leq t \leq \infty}{t}\right) = \frac{1}{\left(\sqrt{2\pi.}\right) V} e^{(-1/2)(t-M/V)^2} \tag{15}$$

Therefore, the probability will be obtained by solving the integral between $-\infty$ and t;

$$P\,(\eta \leq T) = F\,(T) = \frac{1}{\left(\sqrt{2\pi}\right) V} \int_{-\infty}^{t} e^{(-1/2)(t-M/v)^2} \mathrm{d}t \tag{16}$$

Eq. 16 is difficult to integrate, and thus the probability is estimated from an equivalent expression:

$$P\left(\frac{\eta - M}{V} \leq \frac{T - M}{V}\right) = F\left(\frac{T - M}{V}\right) \tag{17}$$

The above expression thus becomes:

$$P\left(\eta' \leq \frac{T - M}{V}\right) = F\left(\frac{T - M}{V}\right) \tag{18}$$

In Eq. 18, η' is a normal variable with a mean of 0 and a variance of 1. This operation is the typification of a normal variable (or estimate of a score z). This strategy makes it easy to calculate the probabilities.

$$z = \frac{T - M}{V} = \frac{x_i - \overline{X}}{S_X} \tag{19}$$

The earliest time of the project's final event is the sum of the mean values of the activity durations which make up the critical path. In other words, the earliest time of the final event will be the mean value of the random variable η which measures the program duration. Furthermore, given that the normal distribution is symmetrical with respect to its mean value M, in a random context the probability of completing a project by the earliest time will be 50%.

The basic application of this mathematical reasoning enables us to determine the number of time units X that are necessary for there to be a given probability β of completing the program. This is valuable information for a project manager to have, and the value of X can be derived from the equation:

$$P\left(\eta \leq X\right) = \beta \tag{20}$$

Typifying the variable gives

$$P\left(\eta' \leq \frac{X - M}{V}\right) = \beta \tag{21}$$

Having found X, normal distribution tables can be used to obtain the abscissa which leaves area β to its left. If this abscissa is Φ, the duration will be given by:

$$\frac{X - M}{V} = \phi \tag{22}$$

or the equivalent expression,

$$X = \phi V + M \tag{23}$$

The variance of an activity duration is an index which estimates the likelihood of being able to carry out the activity in the predicted time. Greater variance means a greater dispersion of times, in other words, less likelihood. Therefore, when there is more than one critical path in a PERT calculation, they will be organized into a hierarchy of 'criticality' on the basis of the corresponding variances.

However, in reality it should not be assumed that the expected duration is the sum of the mean durations of the different activities. In fact, this assumption results in estimates that are lower than the real times achieved.

3 Application of random PERT to intervention programs: an example

Let us consider the case of a physical activity/sports program for elderly people. Table 1 shows the *Priorities Chart*, which comprises four columns. The first includes all the activities of which the project is comprised. The second column shows the activities which precede their counterpart in the first column. The initial activities are identifiable by the fact that they have no preceding activity; the final activities are identifiable as they do not appear in the second column. The third column shows the duration, and the fourth describes the activity (Fig. 1).

Table 1 Priorities chart

Activity	Predecessor	Duration (days)	Description
A	–	14	Program registration
B	–	14	Handing out material
C	A–B	2	Introduction to the program
D	A	24	General mobility activities
E	A	16	Joint mobility activities
F	D	24	Endurance activities Basic aerobics
G	D	6	Basic anaerobic endurance activities
H	G	6	Basic strength activities
I	F	12	Advanced aerobic endurance activities
J	E	24	Activities combining basic aspects
K	C	1	Closing activities
L	H–I–J	1	End of program party

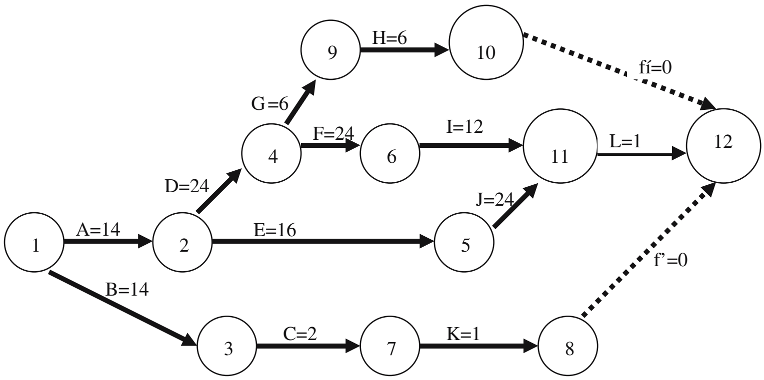

Fig. 1 Graph of the program with numbered nodes and activity durations

Using the term $t(i, j)$ to refer to the time of the activity that links event i with event j, the earliest time will be given by the following expression:

$$t(j) = \max[t(i) + t(i, j)]$$

The minimum time of the program, which indicates its total duration, is given by the value of the earliest time (or latest time) of the project's final event (Fig. 2).

Thus, the earliest times of the PERT in Fig. 1 will be as follows:

1. The earliest time of the program's initial event is zero:
   Node 1: $t(1) = 0$
2. The earliest times of the nodes reached by a single arrow are as follows:
   Node 2: $t(2) = t(1) + t(1, 2) = 0 + 14 = 14$
   Node 3: $t(3) = t(1) + t(1, 3) = 0 + 14 = 14$
   Node 4: $t(4) = t(2) + t(2, 4) = 14 + 24 = 38$
   Node 5: $t(5) = t(2) + t(2, 5) = 14 + 16 = 30$
   Node 6: $t(6) = t(4) + t(4, 6) = 38 + 24 = 62$

Node 7: $t(7) = t(3) + t(3,7) = 14 + 2 = 16$
Node 8: $t(8) = t(7) + t(7,8) = 16 + 1 = 17$
Node 9: $t(9) = t(4) + t(4,9) = 38 + 6 = 44$
Node 10: $t(10) = t(9) + t(9,10) = 44 + 6 = 50$

3. The earliest time of the node reached by two arrows is as follows (only node 11 fulfils this condition):
   Node 11: $t(11) = \max[t(6) + t(6,11), t(5) + t(5,11)] = \max[62 + 12, 30 + 24] = \max[74, 54] = 74$
4. The earliest time of the node reached by three arrows is as follows (only node 12 fulfils this condition):
   Node 12: $t(12) = \max[t(10)+t(10,12), t(11)+t(11,12), t(8)+t(8,12)] = \max[50+0, 74+1, 17+0] = \max[50, 75, 17] = 75$
5. Therefore, the total duration of the program is $t(12) = 75$

Once the earliest times have been calculated we can move on to calculate the latest times. The latest or slow time is the longest time which may be taken in order to reach a given event without delaying the overall program.

The latest time of the program's final event will be equal to the value of its earliest time. For the remaining nodes, and in decreasing order of their assigned number, the latest time will be calculated by taking the lowest value among all the activities related to each node, given by subtracting the time of each activity from its final event time. Therefore, the latest time will be given by the following expression:

$$t(i) = \min[t'(j) + t(i,j)]$$

The latest times of the program will thus be as follows:

1. The latest time of the program's final event is equal to its earliest time:
   Node 12: $t'(12) = 75$
2. The latest times of the nodes reached by a single arrow are as follows:
   Node 11: $t'(11) = t'(12) - t(11,12) = 75 - 1 = 74$
   Node 10: $t'(10) = t'(12) - t(10,12) = 75 - 0 = 75$
   Node 9: $t'(9) = t'(10) - t(9,5) = 75 - 6 = 69$
   Node 8: $t'(8) = t'(12) - t(8,12) = 75 - 0 = 75$
   Node 7: $t'(7) = t'(8) - t(7,8) = 75 - 1 = 74$
   Node 6: $t'(6) = t'(11) - t(6,11) = 74 - 12 = 62$
   Node 5: $t'(5) = t'(11) - t(5,11) = 74 - 24 = 50$
   Node 3: $t'(3) = t'(7) - t(3,7) = 74 - 2 = 72$
3. The latest times of nodes 4 and 2 are:
   Node 4: $t'(4) = \min[t'(9) - t(4,9), t'(6) - t(4,6)] = \min[69-6, 62-24] = \min[63,38] = 38$
   Node 2: $t'(2) = \min[t'(4) - t(2,4), t'(5) - t(2,5)] = \min[38-24, 50-16] = \min[14, 24] = 14$
4. The latest time of node 1 or the initial event will be:
   Node 1: $t'(1) = \min[t'(2) - t(1,2), t'(3) - t(1,3)] = \min[14-14, 72-14] = \min[0, 58] = 0$

Once the various earliest and latest times have been estimated, a final temporal consideration can be made regarding the differences between them; this is known as the slack. The slack refers to the degree of flexibility available according to the early/delayed implementation of certain activities, or the early/delayed completion of others, or the interplay between these (Fig. 3).

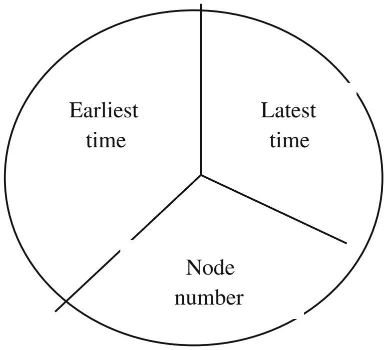

Fig. 2 Representation of the node showing the order number and the earliest and latest times

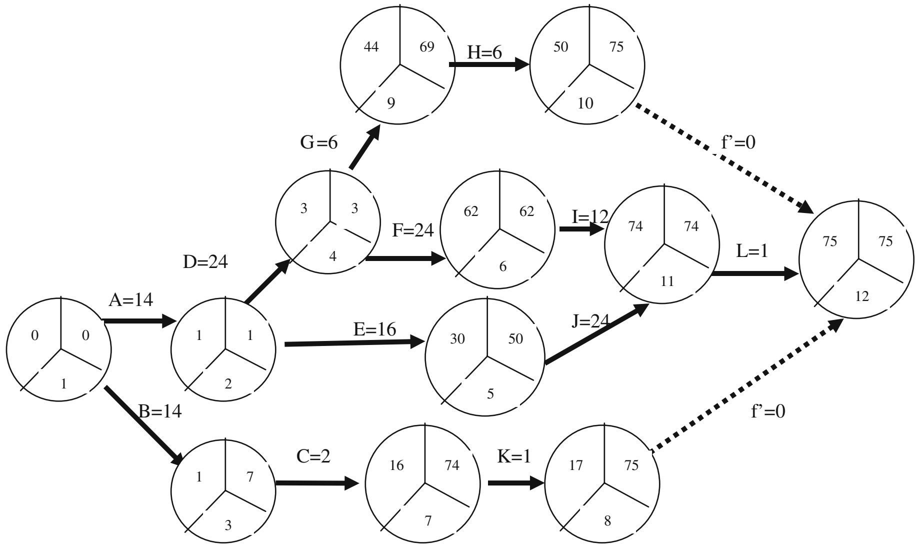

Fig. 3 Graph of the program showing numbered nodes, activity durations and values of earliest and latest times

If we consider that an activity has an initial event and a final event, then the activity will be represented by:

Earliest time of the initial event: $t(i)$

Latest time of the initial event: $t'(i)$

Earliest time of the final event: $t(j)$

Latest time of the final event: $t'(j)$

Two basic kinds of slack can be considered: event slack and activity slack.

1. Event slack: Event slack is the difference between its earliest and latest times. This slack indicates how much delay is permitted in carrying out a given event without delaying the whole program. Within this category two sub-types of slack can be considered:
   1.1 *Start slack*: $H(i)$ is equal to the latest time of the initial event minus the earliest time of the initial event $[H(i) = t'(i) - t(i)]$. This indicates how long the start of an activity may be delayed. If $H(i) = 0$ the start of the activity cannot be delayed.

1.2 *End slack*: $H(j)$ is equal to the latest time of the final event minus the earliest time of the final event $[H(j) = t'(j) - t(j)]$. This indicates how long the end of an activity may be delayed. If $H(j) = 0$ the end of an activity cannot be delayed.

2. Activity slack: This has three types: total activity slack $[H^{\mathrm{T}}(i, j)]$, free slack $[H^{\mathrm{L}}(i, j)]$ and independent slack $[H^{\mathrm{I}}(i, j)]$.

2.1 *Total activity slack*: $[H^{\mathrm{T}}(i, j)]$ is equal to the latest time of the final event minus the initial earliest time minus the activity time.

$$H^{\mathrm{T}}(i, j) = t'(j) - t(i) - t(i, j)$$

This slack indicates how long a given activity can be delayed without delaying the whole program. When the total slack of an activity is equal to zero, it is termed a critical activity. The set of critical activities from the initial program event to the final event is known as the critical path, of which there may be more than one. Critical activities are the key to avoiding delay in the overall program.

2.2 *Free slack*: $[H^L(i, j)]$ is equal to the earliest time of the final event minus the initial earliest time minus the activity time.

$$H^L(i, j) = t(j) - t(i) - t(i, j)$$

This slack indicates that part of the total slack can be used up without affecting subsequent activities.

2.3 *Independent slack*: $[H^I(i, j)]$ is equal to the earliest time of the final event minus the initial latest time minus the activity time.

$$H^I(i, j) = t(j) - t'(i) - t(i, j)$$

This slack indicates the extent to which the total activity slack has been used up.

Critical path: Activities whose total slack is equal to zero are termed critical activities. By linking the critical activities a path is formed from the initial event to the final event, and this is known as the *critical path*. This path is essential for control of the project. The graph shows this path represented by a double line for the activities of which it is comprised. A necessary (although not sufficient) condition for an activity to be critical is that the slack of its initial and final events is equal to zero (Table 2).

On the basis of the Priorities Chart, the sub-graph corresponding to the project's critical path will be as follows (Fig. 4):

The PERT times and variances are as shown, the variances having been calculated according to the formula:

$$Vi^2 = \left(\frac{bi - a}{6}\right)^2$$

Applying the above we can establish that the duration of the project is a normally distributed random variable, with a mean and variance equal to:

$$M = 14 + 24 + 24 + 12 + 1 = 75$$
$$V^2 = 2.32 + 4 + 4 + 2 + 0.16 = 12.48$$

It should be borne in mind that this is stretching the theory somewhat, as we are postulating the convergence to a normal distribution with only five variables.

Table 2 Summary table with all the data of the PERT and critical paths

Activity $i-j$	Identification	Duration $t(_{i,j})$	Earliest start $t(i)$	Earliest finish $t(_j)$	Latest start $t'(i)$	Latest finish $t'(_j)$	Event slack start $H(i)$	Event slack finish $H(_j)$	Total activ. Slack H^T	Free slack H^L	Indep. Slack H^I	Situation
1 – 2	A	14	0	14	0	14	0	0	0	0	0	Critical
1 – 3	B	14	0	14	0	72	0	58	58	0	0	
3 – 7	C	2	14	16	72	74	58	58	58	0		
2 – 4	D	24	14	38	14	38	0	0	0	0	0	Critical
2 – 5	E	16	14	30	14	50	0	20	20	0	0	
4 – 6	F	24	38	62	38	62	0	0	0	0	0	Critical
4 – 9	G	6	38	44	38	69	0	25	25	0	0	
9 – 10	H	6	44	50	69	75	25	25	25	0		
6 – 11	I	12	62	74	62	74	0	0	0	0	0	Critical
5 –11	J	24	30	74	50	74	20	0	20	0	0	
7 – 8	K	1	16	17	74	75	58	58	58	0		
11–12	L	1	74	75	74	75	0	0	0	0	0	Critical
$i-j$		$t(_{i,j})$	$t(i)$	$t(_j)$	$t'(i)$	$t'(_j)$	$H(i)=t'(i)-t(i)$	$H(j)=t'(j)-t(j)$	$H^T(i,j)=t'(j)-t(i)-t(i,j)$	$H^L(i,j)=t(j)-t(i)-t(i,j)$	$H^I(i,j)=t(j)-t'(i)-t(i,j)$	

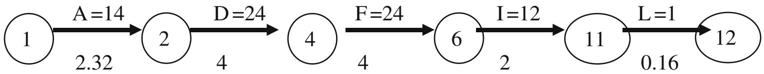

Fig. 4 Critical path of the project

These estimates can be used to consider the likelihood of the program being completed in 100 sessions. This likelihood can be calculated using the following formula:

$$P\left(\eta \leq \frac{100-75}{V}\right) = P(\leq 2.01) = F(2.01)$$

Thus, the likelihood is that situated to the left of the abscissa 2.01 in a normal distribution of mean 0 and variance 1, in this case 0.9555.

4 Discussion

Sport and physical activity are increasingly regarded as characteristic of our society's development and numerous social intervention programs have been planned on this basis, particularly for elderly people. Naturally, these programs must be evaluated. The notion of sport as a genuine mass phenomenon linked to the transmission of an identifiable set of values can be witnessed every day in our streets, parks, gardens, sports centers, gyms and swimming pools—places in which people exercise, run, cycle, swim and walk. Indeed, the leisure society is characterized by the large-scale practice of sport, whose main objective may be health, improved quality of life or the cult of the body.

One of the aims of this study was to illustrate the importance of random PERT and its application to the field of program evaluation, in general, and physical activity/sports programs, in particular. As pointed out above, these programs are linked to the philosophy of social intervention programs.

The study has applied random PERT to a physical activity/sports intervention program for elderly people using simulated data. Starting from the proposed priorities and the predicted durations for each one of the program's activities, the calculation of the earliest and latest times provided information about the different degrees of flexibility or *slack*, and enabled the critical path to be obtained.

However, the specificity of the random PERT procedure in the proposed application has an additional benefit, namely the possibility of knowing how likely it is that the program will be completed in a fixed time period, that is, within a given number of sessions. Access to this information clearly has enormous repercussions in terms of the financial evaluation of intervention programs and, consequently, in determining their efficiency.

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